Instability in slotted wall tunnels

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SUMMARY

A new water tunnel, incorporating a slotted wall working section, was found to suffer from severe vibration. A theoretical explanation is given for this, together with experimental evidence gleaned from this water tunnel and a small wind tunnel. It is shown that the oscillations are hydrodynamic in origin and are associated with the slotted wall design. Consideration is given to methods of elimination or reduction of the oscillations.

1. INTRODUCTION

The slotted wall tunnel is well known as a tool for studying transonic flow. The tunnel choking and interference effects associated with a closed tunnel severely restrict the size of model which can be tested at transonic speeds; on the other hand, open jet tunnels are unsteady and the jet breaks up. The slotted wall working section is intended as a compromise, the solid part of the boundary steadying the flow and the open part of the boundary preventing choking and reducing interference effects.

Even in very subsonic conditions, these advantages are still valid. In a closed tunnel it is not desirable to use a model whose diameter exceeds about one-sixth the working section diameter because of interference effects, and an open jet tunnel is unsuitable if the length of working section is greatly to exceed the diameter of the jet. The replacement of a closed working section by a slotted wall section either enables larger models to be tested (up to about one-third the diameter of the jet) or, at the design stage, permits the use of a smaller tunnel with consequent saving in expense, space and power. However, one present disadvantage of the slotted wall tunnel is the lack of design experience, so that there is an element of risk in embarking on such a design. Until this state of affairs is remedied, it is inevitable that the potential advantages of the slotted wall will remain unrealized.

When new hydroballistic facilities were planned for the Admiralty Research Laboratory, a water tunnel was proposed to permit the testing of torpedo-like bodies of up to 10 in. in diameter. Since this would entail, for a closed tunnel, a working section diameter of 60 in., it was decided to take the risk and to design, instead, a slotted wall working section of diameter 30 in., the length of working section being 15 ft., or 6 diameters. A general

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view of the tunnel is given in figure 1; the total length of the upper limb, which includes the working section, is 71 ft. and would be almost doubled with a 60 in. diameter jet. A full description is given by Lever *et al.* (1957) from which source figure 1 has been adapted. Experience with the tunnel has given much design information, which, it is hoped, will put the design of a slotted wall tunnel on as firm a basis as the design of a closed or open jet tunnel.

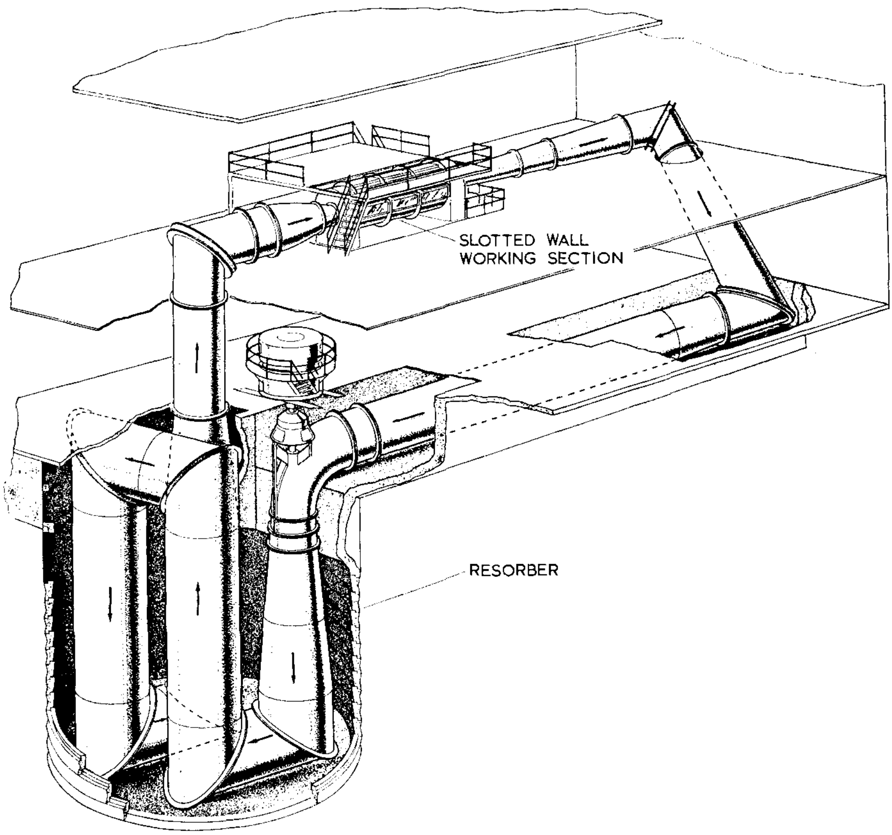

Figure 1. General view of 30 in. water tunnel.

Although the essential requirement is merely that the jet boundary be partly open and partly closed, attention has in the past usually been focused on walls slotted in the direction of flow. A sketch of the essentials of such a working section is given as figure 2. The obvious relevant parameters are the number of slots and the ratio of slot width to slot spacing. The designer, in addition to determining appropriate values of these parameters, also has the task of successfully guiding the jet into the diffuser, the criteria to be satisfied being a constant static pressure distribution along the axis of the working section and sufficiently small wall interference effects on the largest sized model to be tested. A certain amount of work is available on these subjects, both theoretical and experimental (e.g. Wright & Ward (1948); Davis & Moore (1953); Vandrey (1955)), and a small-scale mock-up investigation is appropriate. As a result of such an investigation, the slotted wall parameters and the diffuser dimensions were determined for the A.R.L. tunnel. At the time, it was expected that a satisfactory tunnel would result, particularly since the principles were first applied to a 12 in. diameter tunnel with apparent success. However, from the first, large tunnel vibrations were observed, the entire upper limb being set into motion. A certain reduction was obtained by anchoring the outer shell of the working section but the pressure fluctuations were still excessive both from the point of view of safety and of flow conditions in the working section.

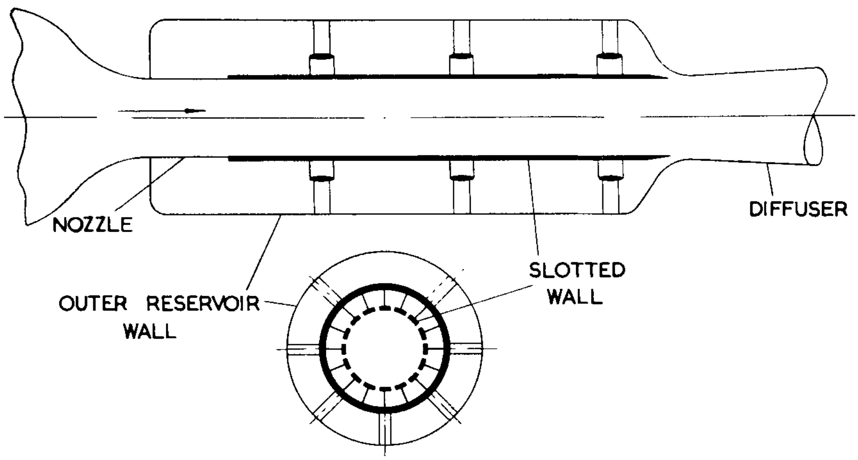

Figure 2. Schematic diagram of a slotted wall working section.

A return was made to the small wind tunnel, which is both more easily instrumented and more quickly modified during test. As a result, the basic explanation of the phenomena occurring was arrived at, but it was found, on investigating the water tunnel further, that certain differences in detail existed between the two tunnels. This paper describes the basic phenomenon, which is the existence of a feed-back loop whereby particular disturbances may be much amplified, draws on experimental evidence from both tunnels to demonstrate this phenomenon, and, finally, describes briefly some possible remedies which have suggested themselves as a result of this work.

2. Outline of mechanism of instability

Self-sustained oscillations in a closed circuit demand that the total gain round the loop be unity and that the phase change round the loop be an integral number of cycles. In practice, the gain exceeds unity for small disturbances, and the amplitude of the final disturbance is determined by non-linearities in the system which ensure that gain is a decreasing function of amplitude. Since losses exist in the circuit, an essential for self-sustained oscillations is that one element of the circuit should be an amplifier, i.e. it should be capable of drawing on a source of energy (in this case, the steady tunnel flow) and converting it to oscillatory energy. This function is performed in the tunnel by the jet itself, bounded by the slotted wall. The remainder of the circuit, the return path, controls the frequency of the oscillation by virtue of the phase change it introduces. Two effective return paths have been found in our investigations; it is considered unlikely that others exist as their attenuation would almost certainly exceed the gain of the amplifier element.

The two elements of the circuit are considered in detail in the later sections of this paper. Before this, their functioning is briefly described.

(*a*) *Amplification by the jet*

It is well known that, if one fluid moves over another with uniform velocity, the common surface may be unstable to certain disturbances (see, for example, Lamb (1932, p. 373)). Particular examples are the generation of waves by wind and the flapping of a flag. If the fluids are of different densities, only disturbances of small wavelength (i.e. high frequency) are unstable, but, if the fluids have the same density, all disturbances are unstable. Surface tension stabilizes small wavelengths so that, with a suitable choice of parameters including the densities of the fluids, both large and small wavelengths may be stable, leaving only a narrow band of frequencies unstable. One frequency in this band will eventually predominate. Similar results apply to a circular jet. If it moves through a fluid of the same density, disturbances of all wavelengths are unstable as is shown by the breaking up of an open jet about one diameter downstream of the nozzle. A slotted wall may be regarded as a stabilizing influence, restraining the radial motion of the jet boundary. The exact mechanism by which the slotted wall restrains the motion is not clear but it will be shown that both large and small wavelengths may be stable. The measured amplification is considerably less than that calculated for an open jet, thus indicating that the slotted wall has a considerable stabilizing influence even although complete stability is not achieved.

(*b*) *The return path*

At entry to the diffuser, the jet contains more oscillatory energy in a certain frequency band than when it left the nozzle. If a return path exists such that energy can be fed back to reinforce the original disturbance for one or more of the frequencies in this band, then large oscillations will occur at these frequencies. One such path, found in the wind tunnel, is formed by the reservoir. Ignoring turbulent mixing, there is no interchange of particles between the jet and the reservoir and only particles which have issued from the nozzle enter the diffuser. The reservoir thus contains a constant mass of fluid whose volume fluctuations due to the oscillations of the jet boundary set up pressure fluctuations.

How these act on the jet leaving the nozzle is not clear; it is possible that they cause the nozzle to vibrate radially. To ensure a phase shift of an integral number of cycles round the loop, it is necessary that the jet fluctuations should produce the maximum pressure in the reservoir when the jet diameter is a minimum at the nozzle, giving the result that the length of the working section must be an integral number of wavelengths less one quarter, a result borne out by experiment. It appears that this mechanism still operates in the water tunnel, but there is now another return path. The water tunnel differs from the wind tunnel in being of closed return type, and pressure fluctuations are transmitted round the return path of the circuit. This results in certain frequencies, approximately independent of water speed, being amplified.

3. Jet instability

Derivation of the basic equations

The cross-section of the tunnel working section is circular. The problem considered is the instability of a circular jet of incompressible fluid, of radius a, moving with velocity U through an infinite extent of the same fluid at rest. The disturbances whose stability is considered are axisymmetric, since only axisymmetric disturbances can operate the feed-back mechanism. (It is assumed that the effect of the slots and gaps of the slotted wall can be averaged round the circumference.) The analysis has been carried out for similar situations by various authors (e.g. Rayleigh 1879), and so is given here only in outline.

The two coordinates of a point are taken as r, the radial distance from the axis, and z, the axial distance from the (arbitrary) origin. At any instant, the common surface of the jet and surrounding fluid is at

$$r = a+\eta,$$

where η is a function of z and t, the time. It is assumed that η is of the first order of smallness; in particular, boundary conditions may be satisfied at $r = a$ rather than at $r = a+\eta$.

Inside the jet, the velocity potential takes the form

$$\Phi = Uz+\phi,$$

and outside

$$\Phi = \phi',$$

where ϕ and ϕ' are the small disturbance potentials associated with η. These potentials satisfy Laplace's equation.

The observed disturbance is a progressive wave moving downstream. Corresponding to the surface disturbance

$$\eta = \eta_0 \exp[i(\sigma t - kz)],$$

we find the solutions

$$\phi = AI_0(kr)\exp[i(\sigma t - kz)],$$

$$\phi' = A'K_0(kr)\exp[i(\sigma t - kz)],$$

having the correct behaviour at $r = 0$ and $r = \infty$ respectively, where I_0

and K_0 are the modified Bessel functions of order zero in the notation of Watson (1944, pp. 77 & 78). (Although the reservoir is not of infinite diameter, calculations show that the simple expression for ϕ' gives numerical results of sufficient accuracy.)

Of the three boundary conditions to be applied at $r = a$, two are obtained by relating the radial component of particle velocity to the motion of the jet boundary. These give

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi'}{\partial r},$$

$$\frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial z} = \frac{\partial \phi}{\partial r},$$

the extra term in the second equation arising from the translational velocity of particles inside the jet. The third condition relates the pressure at the boundary inside and outside the jet. By a mechanism which is discussed later, the slotted wall section is assumed to impose a pressure difference across the common surface, so that

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial z} = \frac{\partial \phi'}{\partial t} - \frac{P}{\rho},$$

where ρ is the density of the fluid, and $P = P_0 \exp[i(\sigma t - kz)]$ is the pressure difference, taken to be positive when the pressure is greater inside the jet.

The boundary conditions give

$$i\sigma\eta_0 - ikU\eta_0 = AkI_0'(ka) = AkI_1(ka),$$

$$i\sigma\eta_0 = A'kK_0'(ka) = -A'kK_1(ka),$$

$$i\sigma AI_0(ka) - i\sigma kUAI_0(ka) = i\sigma A'K_0(ka) - P_0/\rho;$$

and elimination of A and A' from these gives

$$(\sigma - kU)^2 \frac{I_0(ka)}{kI_1(ka)} + \sigma^2 \frac{K_0(ka)}{kK_1(ka)} = \frac{P_0}{\rho\eta_0}.$$

A suitable non-dimensional form of this equation is obtained by the substitutions

$$X = \frac{\sigma}{kU}, \qquad \alpha = \frac{I_1(ka)K_0(ka)}{I_0(ka)K_1(ka)}, \qquad \beta = \frac{I_1(ka)}{kaI_0(ka)}\frac{a}{U^2}\frac{P_0}{\rho\eta_0},$$

which reduce it to

$$(X-1)^2 + \alpha X^2 = \beta. \tag{1}$$

The quantity α is a function of ka only, which increases from zero at $ka = 0$ to unity as $ka \to \infty$. It is plotted in figure 3.

There are two ways of investigating stability. In the situation considered, σ is real and, if (1) gives complex values of X, then k is complex and stability depends on the sign of the imaginary part. Although this represents what happens in practice, it is inconvenient to use this approach as α and β are complicated functions of the complex variable k.

The usual way of investigating stability is to consider an initial sinusoidal surface disturbance and to investigate its behaviour at subsequent times. Then k is taken to be real and (1) is used to determine X as a function of k

for all values of k from zero to infinity. The two cases are related by the assumption that the amplification per cycle in the latter case equals the amplification per wavelength in the former. Although this is not exactly true, it is approximately true provided that the amplification is not too large.

Let a typical root of (1) be $X = X_1 + iX_2$, where X_1 and X_2 are real; then $\sigma = \sigma_1 + i\sigma_2 = kU(X_1 + iX_2)$. All the unknowns η, ϕ, ϕ' and P contain a real exponential factor $\exp(-\sigma_2 t)$ and the motion is therefore unstable when $\sigma_2 < 0$ or $X_2 < 0$.

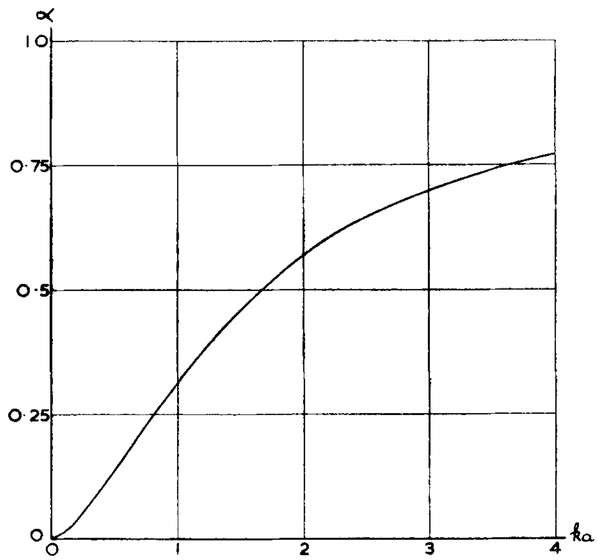

Figure 3. Plot of α *vs* ka.

The measurements taken at a given speed are the frequency f and wavelength λ of the disturbance, and also ϵ, the amplification in a given length l. It follows immediately that

$$k = 2\pi/\lambda, \qquad \sigma_1 = 2\pi f, \tag{2}$$

and, on the assumption that the amplification per wavelength equals the amplification per cycle,

$$\epsilon = \exp(-kl\sigma_2/\sigma_1). \tag{3}$$

The ratio of wave velocity to main jet velocity is

$$f\lambda/U = \sigma_1/kU = X_1.$$

Since α is a function of ka (i.e. $2\pi a/\lambda$), α is known from the wavelength. By means of (2) and (3), k, σ_1 and σ_2 are known and hence X. Thus the entire left-hand side of (1) may be determined from the measurements, so that the effect of a particular slotted wall configuration can be assessed.

The open jet

For an open jet, there is no stabilizing mechanism and $\beta = 0$. Then (1) becomes

$$(X-1)^2 + \alpha X^2 = 0,$$

as was shown by Rayleigh (1879). Since $\alpha > 0$, this admits only the complex solutions

$$X = \frac{1 \pm i\alpha^{1/2}}{1+\alpha},$$

one of which is unstable. The ratio of wave velocity to main jet velocity is

$$X_1 = \frac{1}{1+\alpha}, \tag{4}$$

so that this ratio always lies between 0·5 and 1. The amplification in length l is

$$\epsilon = \exp(kl\alpha^{1/2}). \tag{5}$$

Particular forms of stabilization

To discuss stabilization, it is necessary to consider the term β in (1). This is directly related to P, the pressure difference across the slotted wall, which we postulate to be the mechanism by which the flow is stabilized.

The slotted wall may possibly set up a pressure difference in two ways. To some extent, it may move in and out with the jet boundary. Then it behaves as a skin surrounding the jet and its elastic properties give rise to the pressure discontinuity. Pursuing this analogy, one might expect a term due to surface tension; it would be expected that the bars bending and being pushed against their supports would also give rise to pressure discontinuities of this nature.

The other function of the slotted wall is to permit flow through the gaps. There will then be a pressure difference necessary to overcome the resistance to flow. The total resulting pressure difference is a complicated non-linear function of η and its derivatives, involving, among other things, an unknown relationship between the amplitudes and phases of the motion of the jet boundary and the bars. For small disturbances, this function may be linearized and, although magnitudes cannot be determined, the form of several particular terms in the linear expression can be suggested. The stiffness of the bars on their supports gives a restoring force dependent on the displacement of the bars and so of the jet boundary. Bending of the bars gives a restoring force proportional to the fourth derivative in the axial direction of displacement. Resistance to fluid forced through the gaps gives a term dependent on the outward velocity of the jet boundary. It has been pointed out by Rayleigh (1879) that surface tension if it exists gives rise to two terms, a restoring force proportional to the curvature in the axial direction and a force proportional to displacement which tends to produce divergence from the equilibrium position. Thus we might expect P to contain terms in some or all of η, $\partial^2\eta/\partial z^2$, $\partial^4\eta/\partial z^4$ and $\partial\eta/\partial t$.

Substitution of the sinusoidal oscillations then gives P_0 in the form

$$P_0/\eta_0 = P_1(ka) + iUP_2(ka)X,$$

where it is to be expected from the foregoing considerations that $P_1 \to$ const. as $ka \to 0$, $P_1 \to \infty$ at least as rapidly as $(ka)^2$ as $ka \to \infty$, and $P_2 \propto ka$.

Then

$$\beta = \frac{I_1(ka)}{kaI_0(ka)}\frac{a}{\rho U^2}\{P_1(ka)+iUP_2(ka)X\}$$
$$= 2i\gamma X+\delta, \tag{6}$$

say, where γ and δ are positive functions of ka. Hence (1) becomes

$$X^2(1+\alpha)-2X(1+i\gamma)+1-\delta = 0,$$

the roots of which may be expressed in the form

$$X = \frac{(Z\pm 1)(iY\pm 1)}{1+\alpha}, \tag{7}$$

where

$$\left.\begin{aligned} Y^2-Z^2 &= \gamma^2+\zeta, \\ YZ &= \gamma, \qquad Y, Z > 0, \\ \zeta &= (1+\alpha)(1-\delta)-1. \end{aligned}\right\} \tag{8}$$

From (7) we now have

$$X_1 = \frac{1\pm Z}{1+\alpha}, \qquad X_2 = \pm X_1 Y.$$

An unstable solution only exists if $X_2 < 0$. Thus the upper sign always gives a stable solution and the lower sign gives an unstable solution only if $Z < 1$.

For an unstable solution, the ratio of wave velocity to main jet velocity is $X_1 = (1-Z)/(1+\alpha)$, so that

$$0 < X_1 \leqslant 1/(1+\alpha), \tag{9}$$

and the amplification in length l is

$$\epsilon = \exp(klY). \tag{10}$$

To investigate the range of instability, it is simplest to consider the (γ^2, ζ)-plane. Z is constant along the straight line

$$\gamma^2(Z^{-2}-1)-\zeta = Z^2,$$

and, as is seen in figure 4, the region $0 \leqslant Z < 1$ becomes the region $\zeta > -1$ for all γ^2. The special case $\gamma = 0$ gives stability if $\zeta < 0$ and instability if $\zeta > 0$. The amplification depends on Y, which is constant along the straight line

$$\gamma^2(Y^{-2}+1)+\zeta = Y^2.$$

The line of neutral stability $\zeta = -1$ corresponds to $\delta = 1$.

Several general conclusions can be drawn from the form of β given in (6), making use of properties of the modified Bessel functions given by Watson (1944).

(i) For large wavelengths, i.e. ka small,

$$\gamma \sim Cka/4\rho U,$$

where C is defined by $P_2(ka) = (C/a)ka$,

$$\delta \sim P_1(0)a/2\rho U^2,$$

and $$\alpha \sim \tfrac{1}{2}(ka)^2\log(2/ka).$$

Disturbances are stable if $P_1(0) > 2\rho U^2/a$.

(ii) For small wavelengths, i.e. ka large,

$$\gamma \sim C/2\rho U,$$
$$\delta \sim C'ka/\rho U^2,$$

where C' is defined by $P_1(ka) \sim (C'/a)(ka)^2$, and $C' \to \text{const.} > 0$ or $C' \to \infty$ as $ka \to \infty$, and $\alpha \sim 1$. Disturbances are stable if $C' > \rho U^2/ka$, which is certainly so for large enough values of ka.

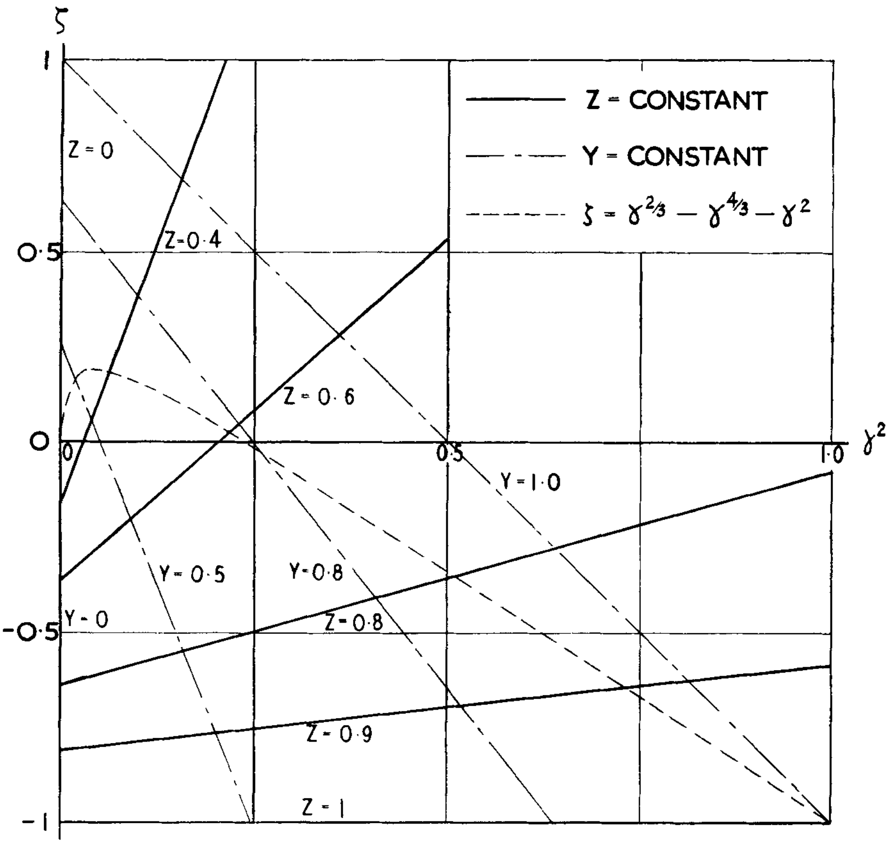

Figure 4. Region of instability in the (γ^2, ζ)-plane $(\zeta > -1)$.

(iii) Even if large and small wavelengths are stable, an intermediate band may not be. Figure 5 shows a particular case where $P_1(ka)$ takes the form $A + B(ka)^4$.

(iv) Increasing the speed of the jet U increases the range of instability, as is also illustrated in figure 5. It is seen that increasing the speed by 10% in the example plotted makes all large wavelengths unstable as well as decreasing the lower limit of unstable wavelengths, and decreasing the speed by 10% stabilizes all wavelengths.

(v) Damping increases the instability of the flow. For a constant wavelength, increasing γ increases Y and so increases both the amplification per unit length and the amplification per cycle. Too much should not be made of this apparently paradoxical result. In the first place, it may be physically unrealistic to suppose that γ can be altered without altering ζ. In the second place, the most relevant measure of instability in this theory is the rate of increase of the disturbance with time. It is seen that the

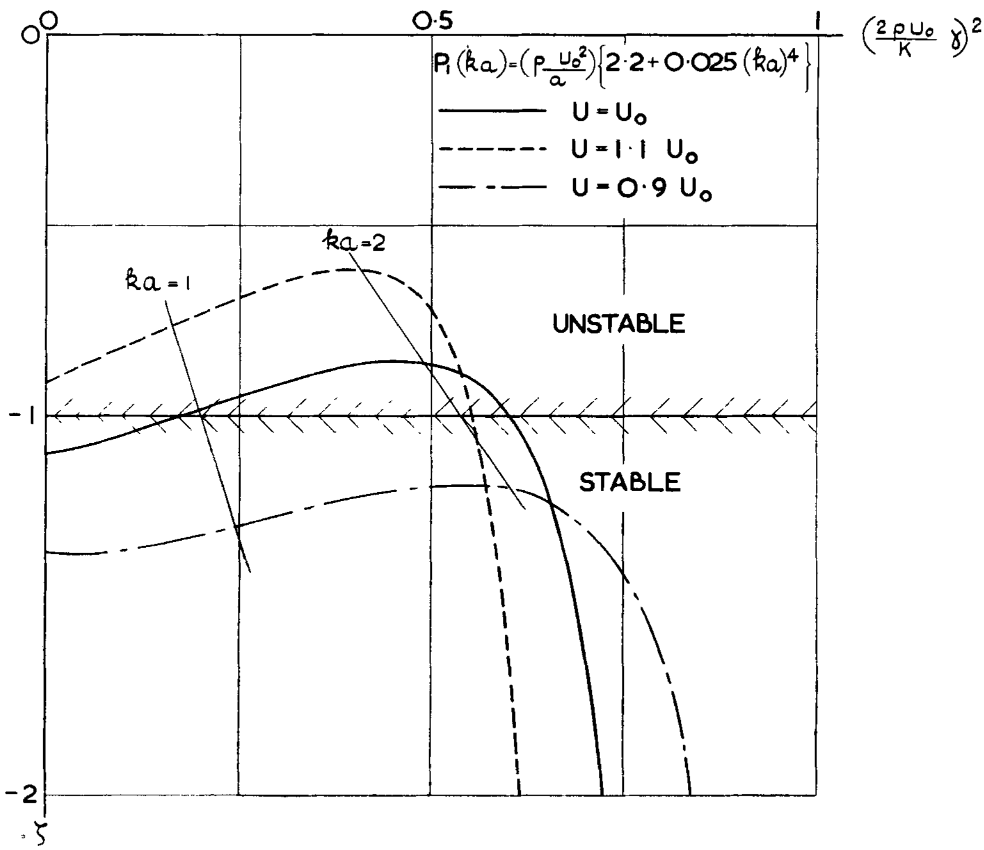

Figure 5. Theoretical example of the dependence of stability on wavelength and main jet velocity.

disturbance increases with time as $\exp\{kUYt(1-Z)/(1+\alpha)\}$; this increases with γ only if $Y^2 < Z$, which is only so inside the curve $\zeta = \gamma^{2/3} - \gamma^{4/3} - \gamma^2$ shown in figure 4. Nevertheless, it is clear that, in some circumstances, the statement is correct, though it should be noted that change of damping cannot destabilize a stable flow.

4. The return path

Quite a broad band of frequencies may be amplified by the jet (as demonstrated in figure 5), and it follows that many paths may exist returning energy from the downstream end of the jet to the nozzle and providing the correct phase relationship for one or more frequency in the band. However, in most cases, the efficiency of the return path is likely to be so low as to reduce the gain round the loop to less than unity, thus eliminating any instability in this loop. It is not possible to consider return paths in any generality, and any new tunnel will present its own problems. Two particular return paths have been identified in the A.R.L. wind and water tunnels, and their occurrence and effect on the tunnels will be described.

Role of the reservoir

Both return paths have one element in common. The boundary of the jet consists of oscillating particle paths leaving the nozzle and re-attaching at some point near entry to the diffuser. It thus encloses a constant mass of fluid in the reservoir which, in the theoretical inviscid, non-oscillating situation would be at rest, separated from the jet by a free streamline. (The effect of viscosity, together with turbulent mixing, is to thicken the jet boundary and to impart momentum in the direction of the jet to the neighbouring fluid in the reservoir. By continuity, this fluid is forced to return along the outer wall of the reservoir so that the flow in the reservoir takes the form of a general swirl set up by an annular ring of vorticity.) The oscillating jet boundary produces a periodic volume fluctuation of the fluid in the reservoir and so an oscillatory pressure. This explanation introduces the compressibility of the fluid. However, this does not have much effect on the amplitude and phase distribution of pressure in the reservoir, as is easily verified by considering the reservoir as a tube closed at the nozzle end and energized by the vibrations of the downstream end of the jet boundary. In all practical cases considered, the tube is well less than a quarter wavelength long, and so the oscillating pressure does not vary greatly in amplitude or phase throughout. This pressure fluctuation must also exist throughout the jet, giving rise to a fluctuation in density; energy must be propagated out of the working section both up and downstream. Rough calculations indicate that, in all cases considered, these pressure fluctuations should completely mask the fluctuations in the jet which are calculable from the theory of the preceding section in the form of amplified progressive waves.

Return path through the reservoir

One return path utilizes the potential energy of these pressure fluctuations in the reservoir. How energy is fed back to the jet is not clear, but it seems probable that the nozzle is set into vibration by the fluctuations. On this assumption it is possible to calculate the preferred frequencies of this circuit.

The displacement of the surface of the jet may be expressed as

$$\eta = \eta_0 \exp[i(\sigma t - kz)],$$

where k is complex. The change in volume of the reservoir is $V = V_0 e^{i\sigma t}$

with

$$V_0 = 2\pi a \eta_0 \int_0^l e^{-ikz}\, dz$$

$$= \frac{2\pi a \eta_0}{k_1 + ik_2} \{e^{k_2 l} \sin k_1 l + i(e^{k_2 l} \cos k_1 l - 1)\},$$

where $k = k_1 + ik_2$, k_1, $k_2 > 0$.

When V takes its maximum value, the reservoir volume is least and so the pressure is greatest. This pressure acts on either the jet or nozzle at the upstream end and it is assumed that at this point the jet diameter is then a minimum. Thus V and η must be an amount π out of phase, so that V_0/η_0 is real and negative. This condition is only satisfied if

$$\frac{e^{k_2 l} \sin k_1 l}{k_1} = \frac{e^{k_2 l} \cos k_1 l - 1}{k_2} < 0.$$

Experimentally, it is known that $e^{k_2 l} \gg 1$ and $k_1 \gg k_2$. Thus $\sin k_1 l \doteqdot -1$, and

$$k_1 l = 2n\pi + 3\pi/2 + \xi, \tag{11}$$

where n is an integer.

Since ξ is small,

$$-\frac{e^{k_2 l}}{k_1} \doteqdot \frac{\xi e^{k_2 l} - 1}{k_2},$$

so that

$$\xi = e^{-k_2 l} - k_2/k_1.$$

Experimental results indicate that $e^{-k_2 l}$ and k_2/k_1 are comparable in magnitude, so that $\xi \doteqdot 0$. Thus, from (11),

$$l/\lambda = n + \tfrac{3}{4}. \tag{12}$$

Return path round the tunnel return circuit

The other return path utilizes the energy propagated out of the working section. It is applicable only to a closed return tunnel when the energy may move round the circuit with relatively little loss. The preferred frequencies are then those for which the time of transit round the tunnel circuit is an integral number of periods.

5. Experimental work

Description of tunnels

(*a*) *Water tunnels*

The 30 in. tunnel is of closed return type as shown in figure 1, and, apart from the working section, is only noteworthy here for its 'resorber'. This feature, whose use in water tunnels originated in the U.S.A., is a portion of the tunnel circuit so located that water is subjected to pressure for long enough to redissolve any air bubbles which may have formed during cavitating flow in the working section. In the present work, its relevance is that it provides an air trap in the circuit halfway round the resorber which materially affects the vibrations observed.

The working section is six diameters long, and the reservoir diameter is two and a half times the jet diameter. The slotted wall consists of sixteen rectangular bars, the gaps between bars comprising 20% of the total boundary. Further details of the tunnel are given by Lever *et al.* (1957). The tunnel was equipped for these tests with static pressure holes in the wall of the reservoir, diffuser and contraction, and Pitot–static tubes in the main jet.

A small amount of work was also done in the A.R.L. 12 in. tunnel. This was originally used as a prototype for the 30 in. tunnel, and is a scaled-down version of it in all important respects, except that it has no resorber.

(*b*) *Wind tunnel*

The 7 in. diameter wind tunnel was designed specifically for the purpose of investigating slotted wall working sections. It is a blow-down tunnel, ending in an open jet to which can be attached any appropriate slotted wall unit. For these investigations, the dimensions of the water tunnel working section and diffuser were scaled down to fit on to the contraction and, after a reasonable length of diffuser, terminated abruptly, giving an open return circuit. The slotted wall was made of Perspex, the outer reservoir wall and the diffuser of acetate sheet, and the nozzle was a Dural extension to the wooden contraction. Some modifications were introduced during tests; in particular, the original contraction from the reservoir to the diffuser was replaced by a ring (see figure 10). These were shaped in wax with an appropriate stiff backing. Measurements were made with hot-wire anemometers, measuring the spectrum of the longitudinal component of turbulence with a frequency analyser whose bandwidth was about 1% of the mid-band frequency. The sensitivity of the hot wires was not known with accuracy, and little use has been made of absolute values. Pressure measurements were also made, a static tube being joined to a microphone. Again absolute values were not known.

Jet instability

The first experiments in the small wind tunnel made use of smoke, and demonstrated that the disturbances were axisymmetric. Using two hot wires, it was possible to demonstrate by comparing phases that the disturbances were progressive waves moving downstream. To determine wavelengths, it was found simpler to add two hot-wire outputs which tend to cancel when the hot wires are spaced a half wavelength apart. Details of these results are presented later; we note here that these measurements indicate the ratio of wave velocity to main jet velocity to be approximately $(1+\alpha)^{-1}$ so that, by (9), $Z \ll 1$.

Removing the outer reservoir wall in this tunnel eliminates the instability, and the spectrum of the flow issuing from the nozzle is fairly 'white'. Comparison with spectra measured downstream gives the amplification along the working section as a function of frequency. A typical example, at a tunnel speed of 75 ft./sec, is given in figure 6 and indicates that at this speed frequencies between 20 and 60 c/s are amplified with a maximum amplification along the length of working section of 6·6 at 40 c/s. If the wavelength is calculated on the assumption that $Z \ll 1$, it is found, using (10), that $Y \ll \alpha^{1/2} < 1$ also. Thus, from (8), $\gamma \ll 1$, $|\zeta| \ll 1$, and so $\delta \doteqdot \alpha/(1+\alpha)$. It is of interest to compare the results with those calculated for an open

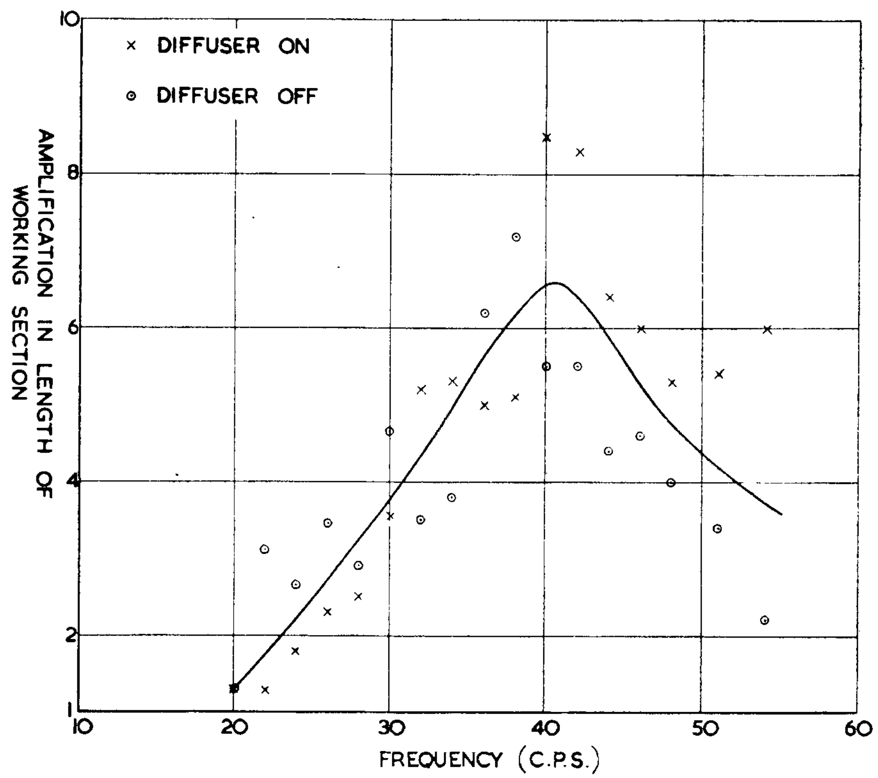

Figure 6. Measured amplification along the jet as a function of frequency for the 7 in. wind tunnel at 75 ft./sec.

jet from (4) and (5). The amplification along the working section for an open jet is found to be a rapidly increasing function of frequency, which at 75 ft./sec, takes the values 13·5 at 20 c/s and 360 at 30 c/s. The effectiveness of the slotted wall in stabilizing the flow is amply demonstrated. The very much less extensive measurements for the 30 in. water tunnel also confirm the tendency towards stabilization.

Return path

(*a*) *Wind tunnel*

The wind tunnel is of open return type. Thus disturbances can only return through the reservoir and the preferred disturbances are those with an integral number of wavelengths less one quarter in the length of the working section.

Since frequency is more easily measured than wavelength, figure 7 (*a*) shows the observed peak frequency or frequencies as a function of tunnel speed. To calculate the expected frequencies, it is assumed that $Z \ll 1$ so that, from (9), the ratio of wave velocity to main jet velocity is $(1+\alpha)^{-1}$.

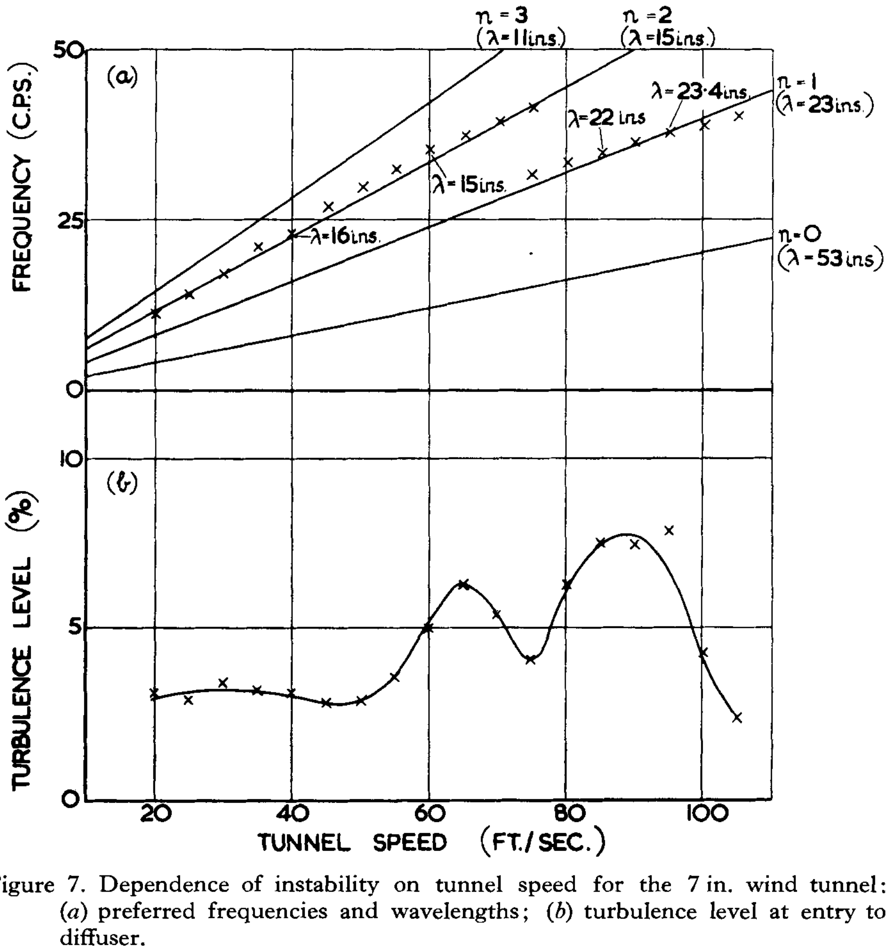

Figure 7. Dependence of instability on tunnel speed for the 7 in. wind tunnel: (*a*) preferred frequencies and wavelengths; (*b*) turbulence level at entry to diffuser.

For each possible wavelength, frequency is then proportional to tunnel speed, and the corresponding straight lines are shown in figure 7 (*a*). The theoretical wavelengths and some of the measured wavelengths are also shown. This leaves no room for doubt that the basic mechanism outlined above is correct and, as previously stated, that the assumption $Z \ll 1$ is justified. The most interesting feature is the jump in frequency at 75 ft./sec, corresponding to a change from two and three-quarters to one and three-quarters wavelengths. Figure 7 (*b*) shows the turbulence level at the diffuser end of the working section as a function of speed. Frequencies in the range about 30 to 40 c/s only are accepted, presumably due to a mechanical resonance associated with this particular tunnel. Below 75 ft./sec, the mode with $n = 1$ (see (12)) is below this frequency range and, above this speed, the mode with $n = 2$ is above it. At 75 ft./sec, both modes are amplified simultaneously and figure 8 (*a*) shows the spectrum at this speed together with that obtained at the same speed with the outer reservoir wall removed. In figure 8 (*b*), a short length of trace is shown taken from

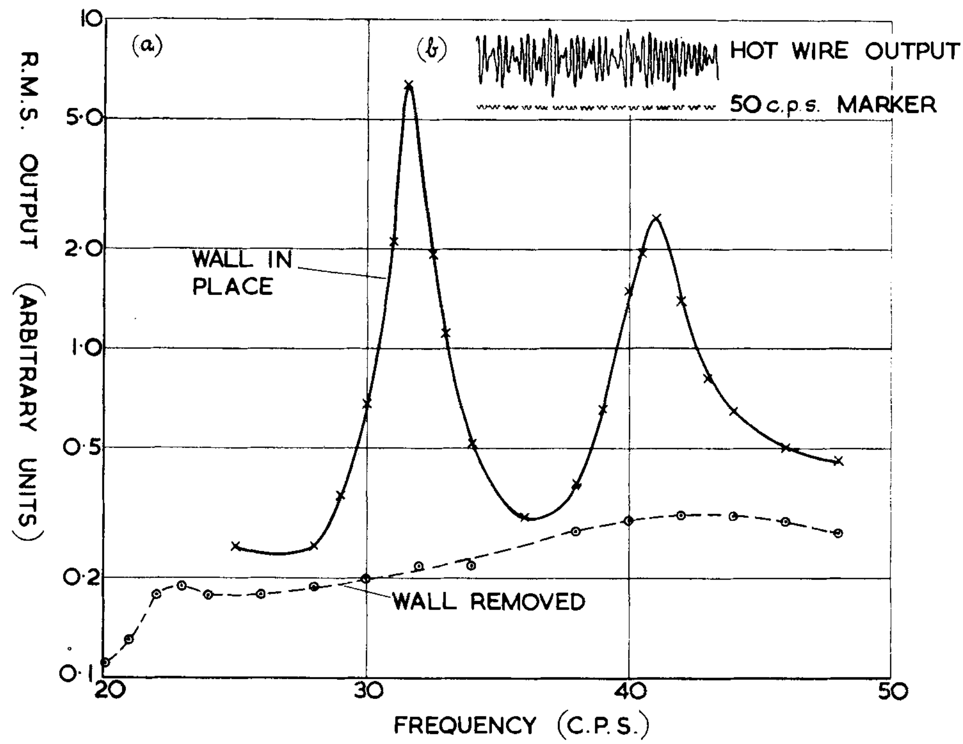

Figure 8. Typical records of turbulence at entry to diffuser at 75 ft./sec. in the 7 in. wind tunnel: (*a*) spectra (i) with outer reservoir wall in place, (ii) with outer reservoir wall removed ; (*b*) cathode-ray oscilloscope record of hot-wire output with outer reservoir wall in place.

a cathode-ray oscilloscope record of the hot-wire output showing the beats produced by the addition of the two frequencies. Since jet instability increases with speed, it is surmised that modes with $n = 3$ and above are not observed since the gain along the jet is too small at the relevant low speeds. The mode with $n = 0$ is never observed since the relevant frequencies only arise above the top speed of the tunnel.

(*b*) *Water tunnels*

The problems of measuring fluctuating pressures and velocities in water are very much greater than those in air. Consequently, although this investigation was aimed at improving the 30 in. water tunnel, the phenomena have been less comprehensively investigated there. The 12 in. water tunnel fares even worse, as its smaller dimensions and the higher frequencies arising increase the difficulties. All that can be said of the 12 in. tunnel is that, although the oscillations were not initially noticed, they were observed when looked for later, and the frequencies observed confirm the existence of the feed-back loop of the wind tunnel.

For the 30 in. tunnel, continuous vibrations such as are obtained in the wind tunnel were rarely obtained, spasmodic bursts being more usually encountered. Frequencies were measured by counting cycles on the oscilloscope traces, and in some cases beats were observed. Although the beat frequency could be estimated, an ambiguity arises as to whether the second frequency is above or below the parent frequency. In the presentation given here, this ambiguity has been resolved simply to fit in best with the general picture presented.

It was discovered—inadvertently in the first instance—that the results were dependent on the amount of air trapped in the resorber. With no air in the resorber, extremely large disturbances were obtained at tunnel speeds between 45 and 60 ft./sec, the shake of the tunnel both being visible and imparting considerable vibrations to the floor. (In the working section, pressure fluctuations up to 9 lb./in.^{2} were observed.) The presence of an air bubble reduced the disturbances to levels which were subjectively acceptable at all speeds. It was therefore surmised that, with no air bubble in the resorber, the return path was by way of the closed return circuit, and the air bubble, when present, acted as a pressure release surface to eliminate this return path.

This surmise is not supported by the frequency measurements, which are presented in figure 9. With air in the resorber (figure 9 (*a*)), the observed frequency is almost independent of water speed being about 6·5 c/s with most observations lying between about 6·2 and 6·8 c/s. With the resorber vented (figure 9 (*b*), which also includes some anomalous results with air in the resorber), the observed frequencies seem to fall into several groups, each increasing with speed although not so rapidly as predicted by theory. Again considerable scatter is observed; this is known to be genuine as different bursts in the same run can reveal marked variations both in frequency and in the strength of observed beats. The observed scatter may be due to the practical difficulty of controlling the amount of air in the resorber.

A theoretical estimate of the time for an acoustic pulse to traverse the return circuit of the tunnel gives 153·2 milliseconds, so that a preferred frequency of 6·53 c/s would be expected for this path. It is clear that, with air in the resorber, this path dominates. With the resorber vented, the amplitude increases and this appears to stimulate the alternative return path through the reservoir. Then, above 40 ft./sec, the two return paths appear to operate in parallel, frequencies between those predicted for the return paths separately being obtained. At the top of the speed range, one or two results suggest that the mode for which $n = 3$ may dominate and impose its frequency on the entire motion.

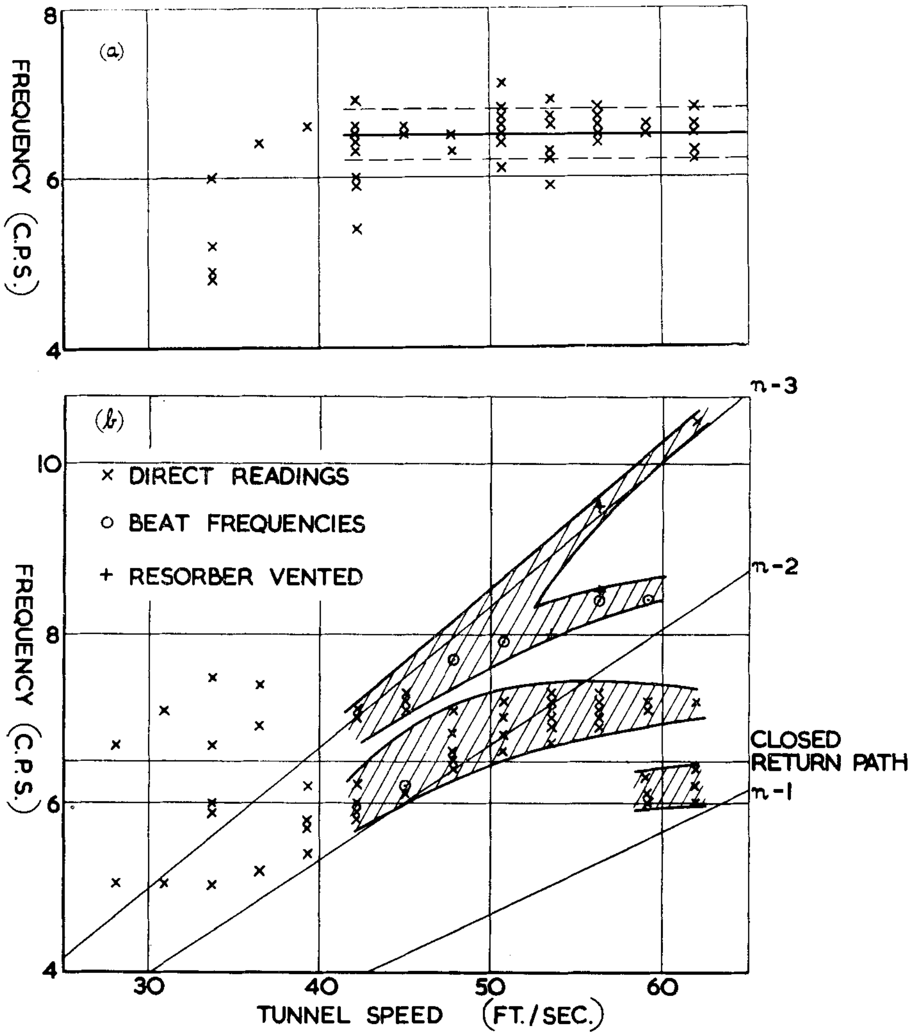

Figure 9. Dependence of preferred frequencies on tunnel speed for the 30 in. water tunnel: (*a*) with air in resorber; (*b*) with resorber vented.

This whole picture suggests that complicated non-linear effects are present (for example, a threshold of instability and frequency entrainment) which are not taken into account by the theory. Nevertheless, it is considered that the evidence does confirm the existence of the two return paths, In both the 30 in. water tunnel and the wind tunnel, the conclusion that the pressure is constant in amplitude and in phase throughout the reservoir and working section is borne out by measurement.

6. Methods of eliminating instability

The purpose of this paper is to give an explanation of the phenomena observed in slotted wall tunnels and to illustrate this with the evidence gathered to date. However, since the understanding of the cause of the oscillations indicates several lines of approach to eliminate them, it is of interest to list these briefly. The degree of success achieved with the various suggested schemes is critically dependent on unknown parameters of the particular tunnel, especially in the dependence of amplitude on the degree of non-linearity of the circuit as well as the gain. For this reason, numerical results are only given to illustrate the discussion, and no great detail is entered into. The particular measures eventually adopted to cure the water tunnel will be reported elsewhere when the investigation is completed.

The first place in which one looks for improvement is the jet itself, the amplifier element which is essential for any instability. The stiffness of the slotted wall can be increased either by stiffening all members or by reducing the gap width. In the wind tunnel, reducing the total gap width from 20% of the circumference to 7% has produced a worthwhile reduction in amplitude. The optimum gap width is likely to be determined by the failure of the section to operate as a slotted wall section, giving model corrections comparable to those of a closed jet. Another remedy lies in the fact that, since the length of the working section is of the order of only two or three wavelengths, end effects may be expected to be significant. This is exemplified in figure 10 which illustrates three different types of entry to the diffuser tested in the water tunnel: viz. a contraction, a ring and a splitter ring. It has been found that the splitter ring is very successful while the ring gives results rather worse than the contraction. The reason for this is not clear. It is conjectured that the jet boundary at re-attachment to the wall is effectively moored by the splitter ring whereas the contraction gives too much freedom of movement to the re-attachment point and the ring to the direction of re-attachment. Whatever the reason, this confirms the point that end effects may be utilized to reduce instability.

Common to both return paths so far observed is the production of large pressure fluctuations in the reservoir. The absolute pressure fluctuations are proportional to the proportional changes in volume of the reservoir, and thus may be reduced by increasing the size of the reservoir. This has not been tested but, as shown in figure 8 (*a*), removing the reservoir wall (thus making its size infinite) completely eliminates the oscillations. A pressure release surface in the reservoir would also have a similar effect. This involves considerable constructional difficulties in the water tunnel; in the wind tunnel the effect has been simulated by drilling holes in the outer reservoir wall. Removal of about $\frac{1}{2}$% of the total wall area was sufficient to reduce the fluctuations to one-tenth of their value. For the two particular return paths, the mechanism by which the pressure fluctuations in the reservoir feed energy back to the nozzle is not yet understood. Accordingly, no remedies can be offered. The efficiency of the return path through the remainder of the closed return circuit can be reduced by providing a pressure release element, as has already been shown.

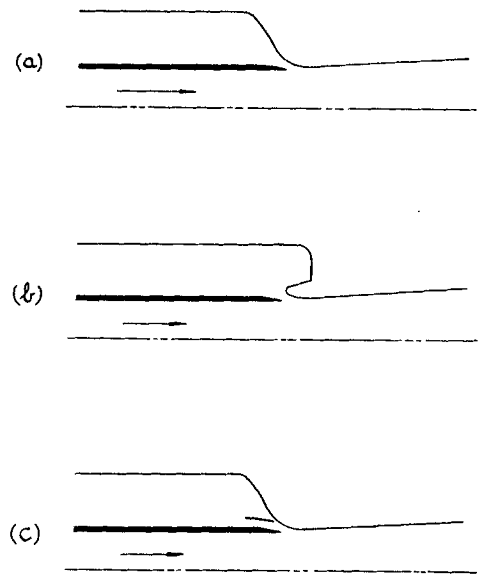

Figure 10. Types of entry to the diffuser tested in the 30 in. water tunnel: (*a*) contraction; (*b*) ring; (*c*) splitter ring.

This section thus provides further evidence that the diagnosis of this paper is correct by showing that deductions from the theory are in accord with experiment.

The authors are indebted to many of their colleagues and particularly to Dr H. Ritter, who was responsible for the tests in the water tunnel and also for several valuable comments.

Appendix. The effect of compressibility on jet instability

In view of the use of the slotted wall section for wind tunnels under transonic conditions, it is of interest to consider the effect of compressibility on jet instability.

As in the incompressible case,

$$u = \frac{\partial \Phi}{\partial r}, \qquad w = \frac{\partial \Phi}{\partial z}, \qquad \text{where } \Phi = Uz + \phi.$$

For small disturbances, we put $\rho = \rho_0 + \rho'$, $p = p_0 + c^2\rho'$, where c is the velocity of sound, and we neglect second-order terms in ϕ and ρ'. The equations of continuity and motion then give

$$\frac{\partial\rho'}{\partial t}+U\frac{\partial\rho'}{\partial z} = -\rho_0\left(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}\right),$$
$$\frac{\partial\phi}{\partial t}+U\frac{\partial\phi}{\partial z} = -\frac{c^2\rho'}{\rho_0}.$$

The substituting of a solution of the form

$$\phi = \phi_0(r)\exp[i(\sigma t-kz)]$$

gives an equation for ϕ_0:

$$\frac{d^2\phi_0}{dr^2}+\frac{1}{r}\frac{d\phi_0}{dr}-k^2\{1-M^2(X-1)^2\}\phi_0 = 0,$$

where M is the free stream Mach number U/c and $X = \sigma/kU$ as before. Using the boundary conditions, we can derive an equation for X in the same form as before:

$$(X-1)^2+\alpha' X^2 = \beta', \tag{1'}$$

where

$$\alpha' = \frac{I_1(k'a)K_0(k'a)}{I_0(k'a)K_1(k'a)},$$
$$\beta' = \frac{k'aI_1(k'a)}{I_0(k'a)}\frac{1}{k^2a^2}\frac{a}{U^2}\frac{P_0}{\rho_0\eta_0},$$
$$k'^2 = k^2\{1-M^2(X-1)^2\}.$$

The appearance of X in α' and β' through its appearance in k' immensely complicates (1′) which is otherwise identical with (1). In particular, we are looking for complex values of X with the result that k' is complex.

When M is small, we can proceed formally and expand in terms of M^2. Taking the first correction term only, we put

$$I_0(k'a) \doteqdot I_0(ka)-\tfrac{1}{2}M^2(X-1)^2kaI_0'(ka)$$

with similar expressions for K_0, I_1 and K_1, so that

$$\alpha' = \alpha+M^2(X-1)^2\bar{\alpha}, \qquad \beta' = \beta+M^2(X-1)^2\bar{\beta},$$

where α and β are the values for the incompressible case, and

$$\frac{\bar{\alpha}}{\alpha} = \tfrac{1}{2}ka\left\{\frac{I_1(ka)}{I_0(ka)}+\frac{K_1(ka)}{K_0(ka)}-\frac{I_0(ka)}{I_1(ka)}-\frac{K_0(ka)}{K_1(ka)}\right\},$$
$$\frac{\bar{\beta}}{\beta} = \tfrac{1}{2}ka\left\{\frac{I_1(ka)}{I_0(ka)}-\frac{I_0(ka)}{I_1(ka)}\right\},$$

in which well-known recurrence relations for Bessel functions have been used. If we replace X by $X+M^2(X-1)^2\bar{X}$, where X is now the incompressible value given by (7), it is found that, to first order,

$$2\bar{X}\{X(1+\alpha)-(1+i\gamma)\} = -\bar{\alpha}X^2+2i\bar{\gamma}X+\bar{\delta}$$
$$= -\bar{\alpha}X^2+(\bar{\beta}/\beta)\{X^2(1+\alpha)-2X+1\},$$

since $\bar{\gamma}/\gamma = \bar{\delta}/\delta = \bar{\beta}/\beta$.

It has been seen in the main part of this paper that, in practice, $Z \ll 1$ and $\gamma \ll Y \ll \alpha^{1/2} < 1$. Then $X = (1-iY)/(1+\alpha)$ for an unstable solution, and

$$-2i\overline{X}(Y+\gamma) = -\frac{\bar{\alpha}}{(1+\alpha)^2}(1-2iY-Y^2)+\frac{\alpha-Y^2}{1+\alpha}\frac{\overline{\beta}}{\beta}$$

or, approximately,

$$\overline{X} = -\frac{\bar{\alpha}}{(1+\alpha)^2}-\frac{i\alpha}{2Y(1+\alpha)^2}\left\{\frac{\bar{\alpha}}{\alpha}-(1+\alpha)\frac{\overline{\beta}}{\beta}\right\}.$$

The real part of X is positive, and the much greater imaginary part is negative. If it can be assumed that $Y^2 \ll \alpha^2$, which is reasonably certain in practice, then it is seen that the effect of compressibility is to increase the amplification of an unstable flow.

The presence of a body in the tunnel affects the flow and the local Mach number may differ greatly from the free stream Mach number. Thus, although at low Mach numbers the presence of a body should only have a small effect on the stability of the tunnel, at higher Mach numbers the effect may be considerable, and a more elaborate theoretical attack does not seem justified.

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